15 Single-phase series a.c. circuits

At the end of this chapter you should be able to:

- draw phasor diagrams and current and voltage waveforms for (a) purely resistive (b) purely inductive and (c) purely capacitive a.c. circuits
- perform calculations involving $X_L = 2\pi f L$ and $X_C = \frac{1}{2\pi f C}$
- draw circuit diagrams, phasor diagrams and voltage and impedance triangles for $R-L$, $R-C$ and $R-L-C$ series a.c. circuits and perform calculations using Pythagoras’ theorem, trigonometric ratios and $Z = \frac{V}{I}$
- understand resonance
- derive the formula for resonant frequency and use it in calculations
- understand Q-factor and perform calculations using
  \[
  V_L (\text{or } V_C) \quad \text{or} \quad \frac{\omega L}{R} \quad \text{or} \quad \frac{1}{\omega C R} \quad \text{or} \quad \frac{1}{R \sqrt{\left( \frac{L}{C} \right)}}
  \]
- understand bandwidth and half-power points
- perform calculations involving $(f_2 - f_1) = \frac{f_r}{Q}$
- understand selectivity and typical values of Q-factor
- appreciate that power $P$ in an a.c. circuit is given by $P = VI \cos \phi$ or $I_R^2 R$ and perform calculations using these formulae
- understand true, apparent and reactive power and power factor and perform calculations involving these quantities

15.1 Purely resistive a.c. circuit

In a purely resistive a.c. circuit, the current $I_R$ and applied voltage $V_R$ are in phase. See Figure 15.1.
15.2 Purely inductive a.c. circuit

In a purely inductive a.c. circuit, the current $I_L$ lags the applied voltage $V_L$ by 90° (i.e. $\pi/2$ rads). See Figure 15.2.

In a purely inductive circuit the opposition to the flow of alternating current is called the inductive reactance, $X_L$

$$X_L = \frac{V_L}{I_L} = 2\pi fL \ \Omega$$

where $f$ is the supply frequency, in hertz, and $L$ is the inductance, in henry’s. $X_L$ is proportional to $f$ as shown in Figure 15.3.

Problem 1. (a) Calculate the reactance of a coil of inductance 0.32 H when it is connected to a 50 Hz supply. (b) A coil has a reactance of 124 Ω in a circuit with a supply of frequency 5 kHz. Determine the inductance of the coil.

(a) Inductive reactance, $X_L = 2\pi fL = 2\pi(50)(0.32) = 100.5 \ \Omega$

(b) Since $X_L = 2\pi fL$, inductance $L = \frac{X_L}{2\pi f} = \frac{124}{2\pi(5000)} = 3.95 \ \text{mH}$

Problem 2. A coil has an inductance of 40 mH and negligible resistance. Calculate its inductive reactance and the resulting current if connected to (a) a 240 V, 50 Hz supply, and (b) a 100 V, 1 kHz supply.

(a) Inductive reactance, $X_L = 2\pi fL = 2\pi(50)(40 \times 10^{-3}) = 12.57 \ \Omega$

Current, $I = \frac{V}{X_L} = \frac{240}{12.57} = 19.09 \ \text{A}$

(b) Inductive reactance, $X_L = 2\pi fL = 2\pi(1000)(40 \times 10^{-3}) = 251.3 \ \Omega$

Current, $I = \frac{V}{X_L} = \frac{100}{251.3} = 0.398 \ \text{A}$

15.3 Purely capacitive a.c. circuit

In a purely capacitive a.c. circuit, the current $I_C$ leads the applied voltage $V_C$ by 90° (i.e. $\pi/2$ rads). See Figure 15.4.

In a purely capacitive circuit the opposition to the flow of alternating current is called the capacitive reactance, $X_C$

$$X_C = \frac{V_C}{I_C} = \frac{1}{2\pi fC} \ \Omega$$

where $C$ is the capacitance in farads.

$X_C$ varies with frequency $f$ as shown in Figure 15.5.
Problem 3. Determine the capacitive reactance of a capacitor of 10 \( \mu \)F when connected to a circuit of frequency (a) 50 Hz (b) 20 kHz.

(a) Capacitive reactance \( X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(10 \times 10^{-6})} \)
\[ = \frac{10^6}{2\pi(50)(10)} = 318.3 \, \Omega \]

(b) \( X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(20 \times 10^3)(10 \times 10^{-6})} = \frac{10^6}{2\pi(20 \times 10^3)(10)} \]
\[ = 0.796 \, \Omega \]

Hence as the frequency is increased from 50 Hz to 20 kHz, \( X_C \) decreases from 318.3 \( \Omega \) to 0.796 \( \Omega \) (see Figure 15.5).

Problem 4. A capacitor has a reactance of 40 \( \Omega \) when operated on a 50 Hz supply. Determine the value of its capacitance.

Since \( X_C = \frac{1}{2\pi f C} \), capacitance \( C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(40)} \, \text{F} \)
\[ = \frac{10^6}{2\pi(50)(40)} \, \mu \text{F} = 79.58 \, \mu \text{F} \]

Problem 5. Calculate the current taken by a 23 \( \mu \)F capacitor when connected to a 240 V, 50 Hz supply.

Current \( I = \frac{V}{X_C} = \frac{V}{\frac{1}{2\pi f C}} = 2\pi f CV = 2\pi(50)(23 \times 10^{-6})(240) \)
\[ = 1.73 \, \text{A} \]

Further problems on purely inductive and capacitive a.c. circuits may be found in Section 15.12, problems 1 to 8, page 234.

15.4 \textit{R–L series a.c. circuit}

In an a.c. circuit containing inductance \( L \) and resistance \( R \), the applied voltage \( V \) is the phasor sum of \( V_R \) and \( V_L \) (see Figure 15.6), and thus the current \( I \) lags the applied voltage \( V \) by an angle lying between 0° and 90° (depending on the values of \( V_R \) and \( V_L \)), shown as angle \( \phi \). In any
a.c. series circuit the current is common to each component and is thus taken as the reference phasor.

From the phasor diagram of Figure 15.6, the ‘voltage triangle’ is derived.

For the \( R–L \) circuit: \( V = \sqrt{(V_R^2 + V_L^2)} \) (by Pythagoras’ theorem)

\[
\tan \phi = \frac{V_L}{V_R} \quad \text{(by trigonometric ratios)}
\]

In an a.c. circuit, the ratio \( \frac{\text{applied voltage}}{\text{current}} \) is called the impedance \( Z \), i.e.

\[
Z = \frac{V}{I} \quad \Omega
\]

If each side of the voltage triangle in Figure 15.6 is divided by current \( I \) then the ‘impedance triangle’ is derived.

For the \( R–L \) circuit: \( Z = \sqrt{(R^2 + X_L^2)} \)

\[
\tan \phi = \frac{X_L}{R} \quad \sin \phi = \frac{X_L}{Z} \quad \cos \phi = \frac{R}{Z}
\]

**Problem 6.** In a series \( R–L \) circuit the p.d. across the resistance \( R \) is 12 V and the p.d. across the inductance \( L \) is 5 V. Find the supply voltage and the phase angle between current and voltage.

From the voltage triangle of Figure 15.6, supply voltage \( V = \sqrt{(12^2 + 5^2)} \) i.e. \( V = 13 \text{ V} \)

(Note that in a.c. circuits, the supply voltage is not the arithmetic sum of the p.d.’s across components. It is, in fact, the phasor sum.)

\[
\tan \phi = \frac{V_L}{V_R} = \frac{5}{12}, \quad \text{from which } \phi = \arctan \left( \frac{5}{12} \right) = 22.62^\circ
\]

\[= 22^\circ 37' \text{ lagging} \]

('Lagging' infers that the current is ‘behind’ the voltage, since phasors revolve anticlockwise.)

**Problem 7.** A coil has a resistance of 4 \( \Omega \) and an inductance of 9.55 mH. Calculate (a) the reactance, (b) the impedance, and (c) the current taken from a 240 V, 50 Hz supply. Determine also the phase angle between the supply voltage and current.

\[ R = 4 \, \Omega; \quad L = 9.55 \, \text{mH} = 9.55 \times 10^{-3} \, \text{H}; \quad f = 50 \, \text{Hz}; \quad V = 240 \, \text{V} \]

(a) Inductive reactance, \( X_L = 2\pi f L = 2\pi(50)(9.55 \times 10^{-3}) = 3 \, \Omega \)
(b) Impedance, \(Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(4^2 + 3^2)} = 5 \Omega\)

(c) Current, \(I = \frac{V}{Z} = \frac{240}{5} = 48 \text{ A}\)

The circuit and phasor diagrams and the voltage and impedance triangles are as shown in Figure 15.6.

Since \(\tan \phi = \frac{X_L}{R} \quad \phi = \arctan \frac{X_L}{R} = \arctan \frac{3}{4} = 36.87^\circ\)

\[= 36^\circ 52' \text{ lagging}\]

**Problem 8.** A coil takes a current of 2 A from a 12 V d.c. supply. When connected to a 240 V, 50 Hz supply the current is 20 A. Calculate the resistance, impedance, inductive reactance and inductance of the coil.

Resistance \(R = \frac{\text{d.c. voltage}}{\text{d.c. current}} = \frac{12}{2} = 6 \Omega\)

Impedance \(Z = \frac{\text{a.c. voltage}}{\text{a.c. current}} = \frac{240}{20} = 12 \Omega\)

Since \(Z = \sqrt{(R^2 + X_L^2)}\), inductive reactance, \(X_L = \sqrt{(Z^2 - R^2)}\)

\[= \sqrt{(12^2 - 6^2)}\]

\[= 10.39 \Omega\]

Since \(X_L = 2\pi f L\), inductance \(L = \frac{X_L}{2\pi f} = \frac{10.39}{2\pi(50)} = 33.1 \text{ mH}\)

This problem indicates a simple method for finding the inductance of a coil, i.e. firstly to measure the current when the coil is connected to a d.c. supply of known voltage, and then to repeat the process with an a.c. supply.

**Problem 9.** A coil of inductance 318.3 mH and negligible resistance is connected in series with a 200 \(\Omega\) resistor to a 240 V, 50 Hz supply. Calculate (a) the inductive reactance of the coil, (b) the impedance of the circuit, (c) the current in the circuit, (d) the p.d. across each component, and (e) the circuit phase angle.

\(L = 318.3 \text{ mH} = 0.3183 \text{ H}; \quad R = 200 \Omega; \quad V = 240 \text{ V}; \quad f = 50 \text{ Hz}\)

The circuit diagram is as shown in Figure 15.6.

(a) Inductive reactance \(X_L = 2\pi f L = 2\pi(50)(0.3183) = 100 \Omega\)

(b) Impedance \(Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(200)^2 + (100)^2} = 223.6 \Omega\)
(c) Current \( I = \frac{V}{Z} = \frac{240}{223.6} = 1.073 \, \text{A} \)

(d) The p.d. across the coil, \( V_L = IX_L = 1.073 \times 100 = 107.3 \, \text{V} \)

The p.d. across the resistor, \( V_R = IR = 1.073 \times 200 = 214.6 \, \text{V} \)

[Check: \( \sqrt{(V_R^2 + V_L^2)} = \sqrt{(214.6)^2 + (107.3)^2} = 240 \, \text{V} \), the supply voltage]

(e) From the impedance triangle, angle \( \phi = \arctan \frac{X_L}{R} = \arctan \left( \frac{100}{200} \right) \)

**Hence the phase angle** \( \phi = 26.57^\circ = 26^\circ34' \) **lagging**

Problem 10. A coil consists of a resistance of 100 \( \Omega \) and an inductance of 200 mH. If an alternating voltage, \( v \), given by \( v = 200 \sin 500t \) volts is applied across the coil, calculate (a) the circuit impedance, (b) the current flowing, (c) the p.d. across the resistance, (d) the p.d. across the inductance and (e) the phase angle between voltage and current.

Since \( v = 200 \sin 500t \) volts then \( V_m = 200 \, \text{V} \) and \( \omega = 2\pi f = 500 \, \text{rad/s} \)

Hence rms voltage \( V = 0.707 \times 200 = 141.4 \, \text{V} \)

Inductive reactance, \( X_L = 2\pi fL = \omega L = 500 \times 200 \times 10^{-3} = 100 \, \text{V} \)

(a) Impedance \( Z = \sqrt{R^2 + X_L^2} = \sqrt{(100^2 + 100^2)} = 141.4 \, \text{V} \)

(b) Current \( I = \frac{V}{Z} = \frac{141.4}{141.4} = 1 \, \text{A} \)

(c) p.d. across the resistance \( V_R = IR = 1 \times 100 = 100 \, \text{V} \)

p.d. across the inductance \( V_L = IX_L = 1 \times 100 = 100 \, \text{V} \)

(e) Phase angle between voltage and current is given by: \( \tan \phi = \frac{X_L}{R} \)

from which, \( \phi = \arctan(100/100) \), hence \( \phi = 45^\circ \) or \( \frac{\pi}{4} \) radians

Problem 11. A pure inductance of 1.273 mH is connected in series with a pure resistance of 30 \( \Omega \). If the frequency of the sinusoidal supply is 5 kHz and the p.d. across the 30 \( \Omega \) resistor is 6 V, determine the value of the supply voltage and the voltage across the 1.273 mH inductance. Draw the phasor diagram.

The circuit is shown in Figure 15.7(a).

Supply voltage, \( V = IZ \)
Current \( I = \frac{V_R}{R} = \frac{6}{30} = 0.20 \text{ A} \)

Inductive reactance \( X_L = 2\pi f L = 2\pi(5 \times 10^{-3})(1.273 \times 10^{-3}) \)

\[ = 40 \Omega \]

Impedance, \( Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(30^2 + 40^2)} = 50 \Omega \)

Supply voltage \( V = IZ = (0.20)(50) = 10 \text{ V} \)

Voltage across the 1.273 mH inductance, \( V_L = IX_L = (0.2)(40) = 8 \text{ V} \)

The phasor diagram is shown in Figure 15.7(b).

(Note that in a.c. circuits, the supply voltage is not the arithmetic sum of the p.d.’s across components but the phasor sum)

Problem 12. A coil of inductance 159.2 mH and resistance 20 Ω is connected in series with a 60 Ω resistor to a 240 V, 50 Hz supply. Determine (a) the impedance of the circuit, (b) the current in the circuit, (c) the circuit phase angle, (d) the p.d. across the 60 Ω resistor and (e) the p.d. across the coil. (f) Draw the circuit phasor diagram showing all voltages.

The circuit diagram is shown in Figure 15.8(a). When impedances are connected in series the individual resistances may be added to give the total circuit resistance. The equivalent circuit is thus shown in Figure 15.8(b).

Inductive reactance \( X_L = 2\pi f L = 2\pi(50)(159.2 \times 10^{-3}) = 50 \Omega \)

(a) Circuit impedance, \( Z = \sqrt{(R_c^2 + X_L^2)} = \sqrt{(80^2 + 50^2)} = 94.34 \Omega \)

(b) Circuit current, \( I = \frac{V}{Z} = \frac{240}{94.34} = 2.544 \text{ A} \)

(c) Circuit phase angle \( \phi = \arctan \left( \frac{X_L}{R} \right) = \arctan \left( \frac{50}{80} \right) = 32^\circ \) lagging

From Figure 15.8(a):

(d) \( V_R = IR = (2.544)(60) = 152.6 \text{ V} \)

(e) \( V_{\text{COIL}} = IZ_{\text{COIL}}, \) where \( Z_{\text{COIL}} = \sqrt{(R_c^2 + X_L^2)} = \sqrt{(20^2 + 50^2)} = 53.85 \Omega \)

Hence \( V_{\text{COIL}} = (2.544)(53.85) = 137.0 \text{ V} \)

(f) For the phasor diagram, shown in Figure 15.9,

\( V_L = IX_L = (2.544)(50) = 127.2 \text{ V} \)
In an a.c. series circuit containing capacitance \( C \) and resistance \( R \), the applied voltage \( V \) is the phasor sum of \( V_R \) and \( V_C \) (see Figure 15.10) and thus the current \( I \) leads the applied voltage \( V \) by an angle lying between 0° and 90° (depending on the values of \( V_R \) and \( V_C \)), shown as angle \( \alpha \).

From the phasor diagram of Figure 15.10, the ‘voltage triangle’ is derived. For the \( R-C \) circuit:

\[
V = \sqrt{(V_R^2 + V_C^2)} \quad \text{(by Pythagoras’ theorem)}
\]

and \( \tan \alpha = \frac{V_C}{V_R} \quad \text{(by trigonometric ratios)} \)

As stated in Section 15.4, in an a.c. circuit, the ratio (applied voltage \( V \))/(current \( I \)) is called the impedance \( Z \), i.e. \( Z = \frac{V}{I} \ \Omega \)

If each side of the voltage triangle in Figure 15.10 is divided by current \( I \) then the ‘impedance triangle’ is derived.

For the \( R-C \) circuit: \( Z = \sqrt{(R^2 + X_C^2)} \)

\[
\tan \alpha = \frac{X_C}{R} \quad \sin \alpha = \frac{X_C}{Z} \quad \text{and} \quad \cos \alpha = \frac{R}{Z}
\]

Problem 13. A resistor of 25 Ω is connected in series with a capacitor of 45 μF. Calculate (a) the impedance, and (b) the current taken from a 240 V, 50 Hz supply. Find also the phase angle between the supply voltage and the current.

\( R = 25 \ \Omega; \ C = 45 \ \mu F = 45 \times 10^{-6} \ \text{F}; \ V = 240 \ \text{V}; \ f = 50 \ \text{Hz} \)

The circuit diagram is as shown in Figure 15.10

Capacitive reactance, \( X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(45 \times 10^{-6})} = 70.74 \ \Omega \)

(a) Impedance \( Z = \sqrt{(R^2 + X_C^2)} = \sqrt{[(25)^2 + (70.74)^2]} = 75.03 \ \Omega \)

(b) Current \( I = \frac{V}{Z} = \frac{240}{75.03} = 3.20 \ \text{A} \)
Phase angle between the supply voltage and current, \( \alpha = \arctan \left( \frac{X_C}{R} \right) \)

hence \( \alpha = \arctan \left( \frac{70.74}{25} \right) = 70.54^\circ = 70^\circ 32' \) leading

('Leading' infers that the current is 'ahead' of the voltage, since phasors revolve anticlockwise.)

Problem 14. A capacitor \( C \) is connected in series with a 40 \( \Omega \) resistor across a supply of frequency 60 Hz. A current of 3 A flows and the circuit impedance is 50 \( \Omega \). Calculate: (a) the value of capacitance, \( C \), (b) the supply voltage, (c) the phase angle between the supply voltage and current, (d) the p.d. across the resistor, and (e) the p.d. across the capacitor. Draw the phasor diagram.

(a) Impedance \( Z = \sqrt{(R^2 + X_C^2)} \)

Hence \( X_C = \sqrt{(Z^2 - R^2)} = \sqrt{(50^2 - 40^2)} = 30 \Omega \)

\( X_C = \frac{1}{2\pi f C} \) hence \( C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)(30)} \) F

\( = 88.42 \mu F \)

(b) Since \( Z = \frac{V}{I} \) then \( V = IZ = (3)(50) = 150 \) V

(c) Phase angle, \( \alpha = \arctan \left( \frac{X_C}{R} \right) = \arctan \left( \frac{30}{40} \right) = 36.87^\circ \)

\( = 36^\circ 52' \) leading

(d) P.d. across resistor, \( V_R = IR = (3)(40) = 120 \) V

(e) P.d. across capacitor, \( V_C = IX_C = (3)(30) = 90 \) V

The phasor diagram is shown in Figure 15.11, where the supply voltage \( V \) is the phasor sum of \( V_R \) and \( V_C \).

Further problems on \( R-C \) a.c. circuits may be found in Section 15.12, problems 14 to 17, page 235.

\[ \text{Figure 15.11} \]

15.6 \( R-L-C \) series a.c. circuit

In an a.c. series circuit containing resistance \( R \), inductance \( L \) and capacitance \( C \), the applied voltage \( V \) is the phasor sum of \( V_R, V_L \) and \( V_C \) (see Figure 15.12). \( V_L \) and \( V_C \) are anti-phase, i.e. displaced by 180°, and there are three phasor diagrams possible — each depending on the relative values of \( V_L \) and \( V_C \)

When \( X_L > X_C \) (Figure 15.12(b)) : \( Z = \sqrt{(R^2 + (X_L - X_C)^2)} \)

and \( \tan \phi = \frac{(X_L - X_C)}{R} \)
When $X_C > X_L$ (Figure 15.12(c)):

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

and

$$\tan \alpha = \frac{(X_C - X_L)}{R}$$

When $X_L = X_C$ (Figure 15.12(d)), the applied voltage $V$ and the current $I$ are in phase. This effect is called **series resonance** (see Section 15.7).

Problem 15. A coil of resistance $5 \ \Omega$ and inductance $120 \ \text{mH}$ in series with a $100 \ \mu\text{F}$ capacitor, is connected to a $300 \ \text{V}, 50 \ \text{Hz}$ supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor.

The circuit diagram is shown in Figure 15.13

$$X_L = 2\pi f L = 2\pi(50)(120 \times 10^{-3}) = 37.70 \ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(100 \times 10^{-6})} = 31.83 \ \Omega$$

Since $X_L$ is greater than $X_C$ the circuit is inductive.

$X_L - X_C = 37.70 - 31.83 = 5.87 \ \Omega$
Impedance \( Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{(5)^2 + (5.87)^2} = 7.71 \, \Omega \)

(a) Current \( I = \frac{V}{Z} = \frac{300}{7.71} = 38.91 \, A \)

(b) Phase angle \( \phi = \arctan \left( \frac{X_L - X_C}{R} \right) = \arctan \left( \frac{5.87}{5} \right) = 49.58^{\circ} \)  
   = 49°35'

(c) Impedance of coil \( Z_{COIL} = \sqrt{(R^2 + X_L^2)} = \sqrt{(5)^2 + (37.70)^2} \)
   = 38.03 \, \Omega

Voltage across coil \( V_{COIL} = IZ_{COIL} = (38.91)(38.03) = 1480 \, V \)

Phase angle of coil = \( \arctan \left( \frac{X_L}{R} \right) = \arctan \left( \frac{37.70}{5} \right) = 82.45^{\circ} \)
   = 82°27' lagging

(d) Voltage across capacitor \( V_C = IX_C = (38.91)(31.83) = 1239 \, V \)

The phasor diagram is shown in Figure 15.14. The supply voltage \( V \) is the phasor sum of \( V_{COIL} \) and \( V_C \)

---

**Series connected impedances**

For series-connected impedances the total circuit impedance can be represented as a single \( L-C-R \) circuit by combining all values of resistance together, all values of inductance together and all values of capacitance together,

\[(\text{remembering that for series connected capacitors } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots).\]

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For example, the circuit of Figure 15.15(a) showing three impedances has an equivalent circuit of Figure 15.15(b).
Problem 16. The following three impedances are connected in series across a 40 V, 20 kHz supply: (i) a resistance of 8 Ω, (ii) a coil of inductance 130 µH and 5 Ω resistance, and (iii) a 10 Ω resistor in series with a 0.25 µF capacitor. Calculate (a) the circuit current, (b) the circuit phase angle and (c) the voltage drop across each impedance.

The circuit diagram is shown in Figure 15.16(a). Since the total circuit resistance is 8 + 5 + 10, i.e. 23 Ω, an equivalent circuit diagram may be drawn as shown in Figure 15.16(b).

Inductive reactance, \( X_L = 2\pi f L = 2\pi(20 \times 10^3)(130 \times 10^{-6}) \)
\[ = 16.34 \Omega \]

Capacitive reactance, \( X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(20 \times 10^3)(0.25 \times 10^{-6})} \)
\[ = 31.83 \Omega \]

Since \( X_C > X_L \), the circuit is capacitive (see phasor diagram in Figure 15.12(c)). \( X_C - X_L = 31.83 - 16.34 = 15.49 \Omega \)

(a) Circuit impedance, \( Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{23^2 + 15.49^2} \)
\[ = 27.73 \Omega \]

Circuit current, \( I = \frac{V}{Z} = \frac{40}{27.73} = 1.442 \text{ A} \)

From Figure 15.12(c), circuit phase angle \( \phi = \arctan \left( \frac{X_C - X_L}{R} \right) \)
\[ i.e., \phi = \arctan \left( \frac{15.49}{23} \right) = 33.96^\circ = 33^\circ 58' \text{ leading} \]

(b) From Figure 15.16(a), \( V_1 = IR_1 = (1.442)(8) = 11.54 \text{ V} \)

\[ V_2 = IZ_2 = I\sqrt{5^2 + 16.34^2} = (1.442)(17.09) = 24.64 \text{ V} \]

\[ V_3 = IZ_3 = I\sqrt{10^2 + 31.83^2} = (1.442)(33.36) = 48.11 \text{ V} \]

The 40 V supply voltage is the phasor sum of \( V_1, V_2 \) and \( V_3 \)

Problem 17. Determine the p.d.'s \( V_1 \) and \( V_2 \) for the circuit shown in Figure 15.17 if the frequency of the supply is 5 kHz. Draw the phasor diagram and hence determine the supply voltage \( V \) and the circuit phase angle.
For impedance $Z_1$:
\[
R_1 = 4 \ \Omega \text{ and } X_L = 2\pi fL = 2\pi(5 \times 10^3)(0.286 \times 10^{-3}) = 8.985 \ \Omega
\]
\[
V_1 = IZ_1 = I\sqrt{(R^2 + X_L^2)} = 5\sqrt{(4^2 + 8.985^2)} = 49.18 \ \text{V}
\]
Phase angle $\phi_1 = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{8.985}{4}\right) = 66°'0'' \text{ lagging}

For impedance $Z_2$:
\[
R_2 = 8 \ \Omega \text{ and } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(5 \times 10^3)(1.273 \times 10^{-6})} = 25.0 \ \Omega
\]
\[
V_2 = IZ_2 = I\sqrt{(R^2 + X_C^2)} = 5\sqrt{(8^2 + 25.0^2)} = 131.2 \ \text{V}
\]
Phase angle $\phi_2 = \arctan\left(\frac{X_C}{R}\right) = \arctan\left(\frac{25.0}{8}\right) = 72°'15'' \text{ leading}

The phasor diagram is shown in Figure 15.18.

The phasor sum of $V_1$ and $V_2$ gives the supply voltage $V$ of 100 V at a phase angle of $53°'0'' \text{ leading}$. These values may be determined by drawing or by calculation — either by resolving into horizontal and vertical components or by the cosine and sine rules.

Further problems on $R$–$L$–$C$ a.c. circuits may be found in Section 15.12, problems 18 to 20, page 235.

15.7 Series resonance

As stated in Section 15.6, for an $R$–$L$–$C$ series circuit, when $X_L = X_C$ (Figure 15.12(d)), the applied voltage $V$ and the current $I$ are in phase. This effect is called series resonance. At resonance:

(i) $V_L = V_C$

(ii) $Z = R$ (i.e. the minimum circuit impedance possible in an $L$–$C$–$R$ circuit)

(iii) $I = \frac{V}{R}$ (i.e. the maximum current possible in an $L$–$C$–$R$ circuit)

(iv) Since $X_L = X_C$, then $2\pi f_L = \frac{1}{2\pi f_C}$

from which, $f_r = \frac{1}{(2\pi)^2LC}$

and, $f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$,

where $f_r$ is the resonant frequency.
(v) The series resonant circuit is often described as an acceptor circuit since it has its minimum impedance, and thus maximum current, at the resonant frequency.

(vi) Typical graphs of current $I$ and impedance $Z$ against frequency are shown in Figure 15.19.

Problem 18. A coil having a resistance of 10 $\Omega$ and an inductance of 125 mH is connected in series with a 60 $\mu$F capacitor across a 120 V supply. At what frequency does resonance occur? Find the current flowing at the resonant frequency.

Resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz

\[
= \frac{1}{2\pi \sqrt{\left(\frac{125}{10^3}\right) \left(\frac{60}{10^6}\right)}} \text{ Hz}
\]

\[
= \frac{1}{2\pi \sqrt{\frac{125 \times 6}{10^6}}} = \frac{1}{2\pi \sqrt{\frac{(125)(6)}{10^4}}}
\]

\[
= \frac{10^4}{2\pi \sqrt{(125)(6)}} = 58.12 \text{ Hz}
\]

At resonance, $X_L = X_C$ and impedance $Z = R$

Hence current, $I = \frac{V}{R} = \frac{120}{10} = 12 \text{ A}$

Problem 19. The current at resonance in a series $L-C-R$ circuit is 100 $\mu$A. If the applied voltage is 2 mV at a frequency of 200 kHz, and the circuit inductance is 50 $\mu$H, find (a) the circuit resistance, and (b) the circuit capacitance.

(a) $I = 100 \mu A = 100 \times 10^{-6} \text{ A}; V = 2 \text{ mV} = 2 \times 10^{-3} \text{ V}$

At resonance, impedance $Z = \text{ resistance } R$

Hence $R = \frac{V}{I} = \frac{2 \times 10^{-3}}{100 \times 10^{-6}} = \frac{2 \times 10^6}{100 \times 10^3} = 20 \Omega$

(b) At resonance $X_L = X_C$

i.e. $2\pi f L = \frac{1}{2\pi f C}$
Hence capacitance
\[ C = \frac{1}{(2\pi f)^2 L} \]
\[ = \frac{1}{(2\pi \times 200 \times 10^3)^2 (50 \times 10^{-6})} \text{F} \]
\[ = \frac{(10^6)(10^6)}{(4\pi^2)(10^{10})(50)} \text{µF} \]
\[ = 0.0127 \text{ µF or 12.7 nF} \]

15.8 Q-factor

At resonance, if \( R \) is small compared with \( X_L \) and \( X_C \), it is possible for \( V_L \) and \( V_C \) to have voltages many times greater than the supply voltage (see Figure 15.12(d)).

\[
\text{Voltage magnification at resonance} = \frac{\text{voltage across } L \text{ (or } C\text{)}}{\text{supply voltage } V}
\]

This ratio is a measure of the quality of a circuit (as a resonator or tuning device) and is called the Q-factor.

Hence Q-factor
\[ \frac{V_L}{V} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{2\pi f, L}{R} \]

Alternatively, Q-factor
\[ \frac{V_C}{V} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{2\pi f, CR} \]

At resonance
\[ f_r = \frac{1}{2\pi \sqrt{(LC)}} \text{ i.e. } 2\pi f_r = \frac{1}{\sqrt{(LC)}} \]

Hence Q-factor
\[ = \frac{2\pi f, L}{R} = \frac{1}{\sqrt{(LC)}} \left( \frac{L}{R} \right) = \frac{1}{R} \sqrt{\left( \frac{L}{C} \right)} \]

(Q-factor is explained more fully in Chapter 28, page 495)

Problem 20. A coil of inductance 80 mH and negligible resistance is connected in series with a capacitance of 0.25 µF and a resistor of resistance 12.5 Ω across a 100 V, variable frequency supply. Determine (a) the resonant frequency, and (b) the current at resonance. How many times greater than the supply voltage is the voltage across the reactances at resonance?

(a) Resonant frequency \( f_r \)

\[ = \frac{1}{2\pi \sqrt{\left[ \frac{80}{10^3} \right] \left( \frac{0.25}{10^6} \right)}} = \frac{1}{2\pi \sqrt{\left( 8 \right)(0.25) \left( 10^8 \right)}} \]
\[
\frac{10^4}{2\pi\sqrt{2}}
= 1125.4 \text{ Hz } = 1.1254 \text{ kHz}
\]

(b) Current at resonance \( I = \frac{V}{R} = \frac{100}{12.5} = 8 \text{ A} \)

Voltage across inductance, at resonance,

\[
V_L = IX_L = (I)(2\pi fL)
= (8)(2\pi)(1125.4)(80 \times 10^{-3})
= 4525.5 \text{ V}
\]

(Also, voltage across capacitor,

\[
V_C = IX_C = \frac{I}{2\pi fC} = \frac{8}{2\pi(1125.4)(0.25 \times 10^{-6})} = 4525.5 \text{ V}
\]

Voltage magnification at resonance \( \frac{V_L}{V} \) or \( \frac{V_C}{V} = \frac{4525.5}{100} = 45.255 \text{ V} \)

i.e. at resonance, the voltage across the reactances are 45.255 times greater than the supply voltage. Hence Q-factor of circuit is 45.255.

Problem 21. A series circuit comprises a coil of resistance 2 \( \Omega \) and inductance 60 mH, and a 30 \( \mu F \) capacitor. Determine the Q-factor of the circuit at resonance.

At resonance, Q-factor

\[
Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{60 \times 10^{-3}}{30 \times 10^{-6}}}
= \frac{1}{2}\sqrt{\frac{60 \times 10^6}{30 \times 10^3}}
= \frac{1}{2}\sqrt{2000} = 22.36
\]

Problem 22. A coil of negligible resistance and inductance 100 mH is connected in series with a capacitance of 2 \( \mu F \) and a resistance of 10 \( \Omega \) across a 50 V, variable frequency supply. Determine (a) the resonant frequency, (b) the current at resonance, (c) the voltages across the coil and the capacitor at resonance, and (d) the Q-factor of the circuit.
Single-phase series a.c. circuits

(a) Resonant frequency, \( f_r = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{\left(\frac{100}{10^3}\right) \left(\frac{2}{10^6}\right)}} \)

\[ = \frac{1}{2\pi \sqrt{\left(\frac{20}{10^8}\right)}} = \frac{1}{2\pi \sqrt{\frac{2\pi \sqrt{20}}{10^4}}} = \frac{10^4}{2\pi \sqrt{20}} \]

\[ = 355.9 \text{ Hz} \]

(b) Current at resonance \( I = \frac{V}{R} = \frac{50}{10} = 5 \text{ A} \)

(c) Voltage across coil at resonance,

\[ V_L = IX_L = I(2\pi f_r L) \]

\[ = (5)(2\pi \times 355.9 \times 100 \times 10^{-3}) \]

\[ = 1118 \text{ V} \]

Voltage across capacitance at resonance,

\[ V_C = IX_C = \frac{I}{2\pi f_r C} \]

\[ = \frac{5}{2\pi (355.9)(2 \times 10^{-6})} \]

\[ = 1118 \text{ V} \]

(d) Q-factor (i.e. voltage magnification at resonance) \( = \frac{V_L}{V} \text{ or } \frac{V_C}{V} \)

\[ = \frac{1118}{50} = 22.36 \]

Q-factor may also have been determined by \( \frac{2\pi f_r L}{R} \text{ or } \frac{1}{2\pi f_r CR} \)

\[ \text{or } \frac{1}{R} \sqrt{\left(\frac{L}{C}\right)} \]

Further problems on series resonance and Q-factor may be found in Section 15.12, problems 21 to 25, page 236.

15.9 Bandwidth and selectivity

Figure 15.20 shows how current \( I \) varies with frequency in an \( R-L-C \) series circuit. At the resonant frequency \( f_r \), current is a maximum value, shown as \( I_r \). Also shown are the points A and B where the current is 0.707 of the maximum value at frequencies \( f_1 \) and \( f_2 \). The power delivered to
the circuit is $I^2R$. At $I = 0.707I_r$, the power is $(0.707I_r)^2R = 0.5I_r^2R$, i.e., half the power that occurs at frequency $f_r$. The points corresponding to $f_1$ and $f_2$ are called the half-power points. The distance between these points, i.e. $(f_2 - f_1)$, is called the bandwidth.

It may be shown that

\[ Q = \frac{f_r}{f_2 - f_1} \quad \text{or} \quad (f_2 - f_1) = f_r \frac{Q}{Q} \]

(This formula is proved in Chapter 28, page 495)

**Problem 23.** A filter in the form of a series $L$–$R$–$C$ circuit is designed to operate at a resonant frequency of 5 kHz. Included within the filter is a 20 mH inductance and 10 $\Omega$ resistance. Determine the bandwidth of the filter.

Q-factor at resonance is given by

\[ Q_r = \frac{\omega_0 L}{R} = \frac{(2\pi 5000)(20 \times 10^{-3})}{10} = 62.83 \]

Since $Q_r = f_r/(f_2 - f_1)$

\[ \text{bandwidth, } (f_2 - f_1) = f_r \frac{Q}{Q_r} = 5000 \frac{62.83}{62.83} = 79.6 \text{ Hz} \]

**Selectivity** is the ability of a circuit to respond more readily to signals of a particular frequency to which it is tuned than to signals of other frequencies. The response becomes progressively weaker as the frequency departs from the resonant frequency. The higher the Q-factor, the narrower the bandwidth and the more selective is the circuit. Circuits having high Q-factors (say, in the order of 100 to 300) are therefore useful in communications engineering. A high Q-factor in a series power circuit has disadvantages in that it can lead to dangerously high voltages across the insulation and may result in electrical breakdown.

(For more on bandwidth and selectivity see Chapter 28, page 504)

### 15.10 Power in a.c. circuits

In Figures 15.21(a)–(c), the value of power at any instant is given by the product of the voltage and current at that instant, i.e. the instantaneous power, $p = vi$, as shown by the broken lines.

(a) For a purely resistive a.c. circuit, the average power dissipated, $P$, is given by:

\[ P = VI = I^2R = \frac{V^2}{R} \text{ watts} \] (V and $I$ being rms values).

See Figure 15.21(a).
(b) For a purely inductive a.c. circuit, the average power is zero. See Figure 15.21(b).

(c) For a purely capacitive a.c. circuit, the average power is zero. See Figure 15.21(c).

Figure 15.22 shows current and voltage waveforms for an $R-L$ circuit where the current lags the voltage by angle $\phi$. The waveform for power (where $p = vi$) is shown by the broken line, and its shape, and hence average power, depends on the value of angle $\phi$.

For an $R-L$, $R-C$ or $R-L-C$ series a.c. circuit, the average power $P$ is given by:

$$P = VI \cos \phi \text{ watts}$$

or

$$P = I^2R \text{ watts}$$  (V and I being rms values)

The formulae for power are proved in Chapter 26, page 459.

Problem 24. An instantaneous current, $i = 250 \sin 60t$ mA flows through a pure resistance of 5 k$\Omega$. Find the power dissipated in the resistor.

Power dissipated, $P = I^2R$ where $I$ is the rms value of current.

If $i = 250 \sin 60t$ mA, then $I_m = 0.250$ A and rms current, $I = (0.707 \times 0.250)$ A

Hence power $P = (0.707 \times 0.250)^2(5000) = 156.2$ watts

Problem 25. A series circuit of resistance 60 $\Omega$ and inductance 75 m$H$ is connected to a 110 V, 60 Hz supply. Calculate the power dissipated.

Inductive reactance, $X_L = 2\pi fL = 2\pi(60)(75 \times 10^{-3}) = 28.27$ $\Omega$

Impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{(60)^2 + (28.27)^2} = 66.33$ $\Omega$

Current, $I = \frac{V}{Z} = \frac{110}{66.33} = 1.658$ A

To calculate power dissipation in an a.c. circuit two formulae may be used:

(i) $P = I^2R = (1.658)^2(60) = 165$ W

or (ii) $P = VI \cos \phi$ where $\cos \phi = \frac{R}{Z} = \frac{60}{66.33} = 0.9046$

Hence $P = (110)(1.658)(0.9046) = 165$ W
15.11 Power triangle and power factor

Figure 15.23(a) shows a phasor diagram in which the current I lags the applied voltage V by angle φ. The horizontal component of V is V cos φ and the vertical component of V is V sin φ. If each of the voltage phasors is multiplied by I, Figure 15.23(b) is obtained and is known as the ‘power triangle’.

\[
\begin{align*}
\text{Apparent power,} & \quad S = VI \text{ voltamperes (VA)} \\
\text{True or active power,} & \quad P = VI \cos \phi \text{ watts (W)} \\
\text{Reactive power,} & \quad Q = VI \sin \phi \text{ reactive voltamperes (var)}
\end{align*}
\]

\[
\text{Power factor} = \frac{\text{True power } P}{\text{Apparent power } S}
\]

For sinusoidal voltages and currents, power factor \( \frac{P}{S} = \frac{VI \cos \phi}{VI} \), i.e.

\[
p.f. = \cos \phi = \frac{R}{Z} \quad \text{(from Figure 15.6)}
\]

The relationships stated above are also true when current I leads voltage V. More on the power triangle and power factor is contained in Chapter 26, page 464.

Problem 26. A pure inductance is connected to a 150 V, 50 Hz supply, and the apparent power of the circuit is 300 VA. Find the value of the inductance.

Apparent power \( S = VI \)

Hence current \( I = \frac{S}{V} = \frac{300}{150} = 2 \) A

Inductive reactance \( X_L = \frac{V}{I} = \frac{150}{2} = 75 \) Ω

Since \( X_L = 2\pi fL \), inductance \( L = \frac{X_L}{2\pi f} = \frac{75}{2\pi(50)} = 0.239 \) H

Problem 27. A transformer has a rated output of 200 kVA at a power factor of 0.8. Determine the rated power output and the corresponding reactive power.

\[ VI = 200 \text{ kVA} = 200 \times 10^3; \quad \text{p.f.} = 0.8 = \cos \phi \]

Power output, \( P = VI \cos \phi = (200 \times 10^3)(0.8) = 160 \text{ kW} \)
Reactive power, \( Q = VI \sin \phi \)
If \( \cos \phi = 0.8 \), then \( \phi = \arccos 0.8 = 36.87^\circ = 36'52' \)
Hence \( \sin \phi = \sin 36.87^\circ = 0.6 \)
Hence reactive power, \( Q = (200 \times 10^3)(0.6) = 120 \text{ kvar} \)

**Problem 28.** The power taken by an inductive circuit when connected to a 120 V, 50 Hz supply is 400 W and the current is 8 A. Calculate (a) the resistance, (b) the impedance, (c) the reactance, (d) the power factor, and (e) the phase angle between voltage and current.

(a) Power \( P = I^2R \). Hence \( R = \frac{P}{I^2} = \frac{400}{(8)^2} = 6.25 \ \Omega \)

(b) Impedance \( Z = \frac{V}{I} = \frac{120}{8} = 15 \ \Omega \)

(c) Since \( Z = \sqrt{R^2 + X_L^2} \), then \( X_L = \sqrt{(Z^2 - R^2)} \)
\( = \sqrt{(15)^2 - (6.25)^2} \)
\( = 13.64 \ \Omega \)

(d) Power factor = \( \frac{\text{true power}}{\text{apparent power}} = \frac{VI \cos \phi}{VI} = \frac{400}{(120)(8)} = 0.4167 \)

(c) p.f. = \( \cos \phi = 0.4167 \). Hence phase angle \( \phi = \arccos 0.4167 \)
\( = 65.37^\circ \)
\( = 65^\circ 22' \text{ lagging} \)

**Problem 29.** A circuit consisting of a resistor in series with a capacitor takes 100 watts at a power factor of 0.5 from a 100 V, 60 Hz supply. Find (a) the current flowing, (b) the phase angle, (c) the resistance, (d) the impedance, and (e) the capacitance.

(a) Power factor = \( \frac{\text{true power}}{\text{apparent power}} \)
\( \text{i.e.} \ 0.5 = \frac{100}{(100)(I)} \). Hence \( I = \frac{100}{(0.5)(100)} = 2 \ \text{A} \)

(b) Power factor = 0.5 = \( \cos \phi \). Hence phase angle \( \phi = \arccos 0.5 \)
\( = 60^\circ \text{ leading} \)

(c) Power \( P = I^2R \). Hence resistance \( R = \frac{P}{I^2} = \frac{100}{(2)^2} = 25 \ \Omega \)

(d) Impedance \( Z = \frac{V}{I} = \frac{100}{2} = 50 \ \Omega \)
(e) Capacitive reactance, \( X_C = \sqrt{(Z^2 - R^2)} = \sqrt{(50^2 - 25^2)} \)

\[
X_C = \frac{1}{2\pi f C} \text{ hence capacitance } C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)(43.30)} \text{ F}
\]

\[
= 61.26 \mu \text{F}
\]

Further problems on power in a.c. circuits may be found in Section 15.12 following, problems 26 to 36, page 237.

15.12 Further problems on single-phase series a.c. circuits

A.c. circuits containing pure inductance and pure capacitance

1. Calculate the reactance of a coil of inductance 0.2 H when it is connected to (a) a 50 Hz, (b) a 600 Hz and (c) a 40 kHz supply.

   [(a) 62.83 Ω (b) 754 Ω (c) 50.27 kΩ]

2. A coil has a reactance of 120 Ω in a circuit with a supply frequency of 4 kHz. Calculate the inductance of the coil.

   [4.77 mH]

3. A supply of 240 V, 50 Hz is connected across a pure inductance and the resulting current is 1.2 A. Calculate the inductance of the coil.

   [0.637 H]

4. An e.m.f. of 200 V at a frequency of 2 kHz is applied to a coil of pure inductance 50 mH. Determine (a) the reactance of the coil, and (b) the current flowing in the coil.

   [(a) 628 Ω (b) 0.318 A]

5. Calculate the capacitive reactance of a capacitor of 20 µF when connected to an a.c. circuit of frequency (a) 20 Hz, (b) 500 Hz, (c) 4 kHz

   [(a) 397.9 Ω (b) 15.92 Ω (c) 1.989 Ω]

6. A capacitor has a reactance of 80 Ω when connected to a 50 Hz supply. Calculate the value of its capacitance.

   [39.79 µF]

7. A capacitor has a capacitive reactance of 400 Ω when connected to a 100 V, 25 Hz supply. Determine its capacitance and the current taken from the supply.

   [15.92 µF, 0.25 A]

8. Two similar capacitors are connected in parallel to a 200 V, 1 kHz supply. Find the value of each capacitor if the circuit current is 0.628 A.

   [0.25 µF]

R–L a.c. circuits

9. Determine the impedance of a coil which has a resistance of 12 Ω and a reactance of 16 Ω

   [20 Ω]

10. A coil of inductance 80 mH and resistance 60 Ω is connected to a 200 V, 100 Hz supply. Calculate the circuit impedance and the
current taken from the supply. Find also the phase angle between the current and the supply voltage.

\[ 78.27 \ \Omega, \ 2.555 \ \text{A}, \ 39°57′ \text{ lagging} \]

11 An alternating voltage given by \( v = 100 \sin 240t \) volts is applied across a coil of resistance 32 \( \Omega \) and inductance 100 mH. Determine (a) the circuit impedance, (b) the current flowing, (c) the p.d. across the resistance, and (d) the p.d. across the inductance.

\[(a) \ 40 \ \Omega \ (b) \ 1.77 \ \text{A} \ (c) \ 56.64 \ \text{V} \ (d) \ 42.48 \ \text{V}\]

12 A coil takes a current of 5 A from a 20 V d.c. supply. When connected to a 200 V, 50 Hz a.c. supply the current is 25 A. Calculate the (a) resistance, (b) impedance and (c) inductance of the coil.

\[(a) \ 4 \ \Omega \ (b) \ 8 \ \Omega \ (c) \ 22.05 \ \text{mH}\]

13 A coil of inductance 636.6 mH and negligible resistance is connected in series with a 100 \( \Omega \) resistor to a 250 V, 50 Hz supply. Calculate (a) the inductive reactance of the coil, (b) the impedance of the circuit, (c) the current in the circuit, (d) the p.d. across each component, and (e) the circuit phase angle.

\[(a) \ 200 \ \Omega \ (b) \ 223.6 \ \Omega \ (c) 1.118 \ \text{A} \\
\quad \ (d) \ 223.6 \ \text{V}, \ 111.8 \ \text{V} \ (e) \ 63°26′ \text{ lagging}\]

**R–C a.c. circuits**

14 A voltage of 35 V is applied across a \( C–R \) series circuit. If the voltage across the resistor is 21 V, find the voltage across the capacitor.

\[ 28 \ \text{V} \]

15 A resistance of 50 \( \Omega \) is connected in series with a capacitance of 20 \( \mu\text{F} \). If a supply of 200 V, 100 Hz is connected across the arrangement find (a) the circuit impedance, (b) the current flowing, and (c) the phase angle between voltage and current.

\[(a) \ 93.98 \ \Omega \ (b) \ 2.128 \ \text{A} \ (c) \ 57°51′ \text{ leading}\]

16 An alternating voltage \( v = 250 \sin 800 t \) volts is applied across a series circuit containing a 30 \( \Omega \) resistor and 50 \( \mu\text{F} \) capacitor. Calculate (a) the circuit impedance, (b) the current flowing, (c) the p.d. across the resistor, (d) the p.d. across the capacitor, and (e) the phase angle between voltage and current.

\[(a) \ 39.05 \ \Omega \ (b) \ 4.527 \ \text{A} \ (c) \ 135.8 \ \text{V} \\
\quad \ (d) \ 113.2 \ \text{V} \ (e) \ 39°48′\]

17 A 400 \( \Omega \) resistor is connected in series with a 2358 pF capacitor across a 12 V a.c. supply. Determine the supply frequency if the current flowing in the circuit is 24 mA.

\[ 225 \ \text{kHz} \]

**R–L–C a.c. circuits**

18 A 40 \( \mu\text{F} \) capacitor in series with a coil of resistance 8 \( \Omega \) and inductance 80 mH is connected to a 200 V, 100 Hz supply. Calculate (a) the circuit impedance, (b) the current flowing, (c) the phase angle
between voltage and current, (d) the voltage across the coil, and 
ed (e) the voltage across the capacitor.

[(a) 13.18 Ω (b) 15.17 A (c) 52°38’ 
ed (d) 772.1 V (e) 603.6 V]

19 Three impedances are connected in series across a 100 V, 2 kHz 
supply. The impedances comprise:

(i) an inductance of 0.45 mH and 2 Ω resistance,
(ii) an inductance of 570 µH and 5 Ω resistance, and
(iii) a capacitor of capacitance 10 µF and resistance 3 Ω.

Assuming no mutual inductive effects between the two inductances 
calculate (a) the circuit impedance, (b) the circuit current, (c) the 
circuit phase angle and (d) the voltage across each impedance. Draw 
the phasor diagram.

[(a) 11.12 Ω (b) 8.99 A (c) 25°55’ lagging 
ed (d) 53.92 V, 78.53 V, 76.46 V]

20 For the circuit shown in Figure 15.24 determine the voltages $V_1$ and 
$V_2$ if the supply frequency is 1 kHz. Draw the phasor diagram and 
hence determine the supply voltage $V$ and the circuit phase angle. 

$[V_1 = 26.0 \text{ V}, V_2 = 67.05 \text{ V}, 
V = 50 \text{ V}, 53°8’ \text{ leading}]$

**Figure 15.24**

**Series resonance and Q-factor**

21 Find the resonant frequency of a series a.c. circuit consisting of a coil 
of resistance 10 Ω and inductance 50 mH and capacitance 0.05 µF. 
Find also the current flowing at resonance if the supply voltage is 
100 V. 

$[3.183 \text{ kHz}, 10 \text{ A}]$

22 The current at resonance in a series $L-C-R$ circuit is 0.2 mA. If the 
applied voltage is 250 mV at a frequency of 100 kHz and the circuit 
capacitance is 0.04 µF, find the circuit resistance and inductance. 

$[1.25 \text{ kΩ}, 63.3 \text{ µH}]$

23 A coil of resistance 25 Ω and inductance 100 mH is connected 
in series with a capacitance of 0.12 µF across a 200 V, variable 
frequency supply. Calculate (a) the resonant frequency, (b) the 
current at resonance and (c) the factor by which the voltage across 
the reactance is greater than the supply voltage. 

$[(a) 1.453 \text{ kHz} (b) 8 \text{ A} (c) 36.52]$

24 Calculate the inductance which must be connected in series with a 
1000 pF capacitor to give a resonant frequency of 400 kHz. 

$[0.158 \text{ mH}]$

25 A series circuit comprises a coil of resistance 20 Ω and inductance 
2 mH and a 500 pF capacitor. Determine the Q-factor of the circuit at 
resonance. If the supply voltage is 1.5 V, what is the voltage across 
the capacitor? 

$[100, 150 \text{ V}]$
Power in a.c. circuits

26 A voltage \( v = 200 \sin \omega t \) volts is applied across a pure resistance of 1.5 k\( \Omega \). Find the power dissipated in the resistor. [13.33 W]

27 A 50 \( \mu \)F capacitor is connected to a 100 V, 200 Hz supply. Determine the true power and the apparent power. [0, 628.3 VA]

28 A motor takes a current of 10 A when supplied from a 250 V a.c. supply. Assuming a power factor of 0.75 lagging find the power consumed. Find also the cost of running the motor for 1 week continuously if 1 kWh of electricity costs 7.20 p. [1875 W, £22.68]

29 A motor takes a current of 12 A when supplied from a 240 V a.c. supply. Assuming a power factor of 0.75 lagging, find the power consumed. [2.16 kW]

30 A substation is supplying 200 kVA and 150 kvar. Calculate the corresponding power and power factor. [132 kW, 0.66]

31 A load takes 50 kW at a power factor of 0.8 lagging. Calculate the apparent power and the reactive power. [62.5 kVA, 37.5 kvar]

32 A coil of resistance 400 \( \Omega \) and inductance 0.20 H is connected to a 75 V, 400 Hz supply. Calculate the power dissipated in the coil. [5.452 W]

33 An 80 \( \Omega \) resistor and a 6 \( \mu \)F capacitor are connected in series across a 150 V, 200 Hz supply. Calculate (a) the circuit impedance, (b) the current flowing and (c) the power dissipated in the circuit.

(a) 154.9 \( \Omega \) (b) 0.968 A (c) 75 W]

34 The power taken by a series circuit containing resistance and inductance is 240 W when connected to a 200 V, 50 Hz supply. If the current flowing is 2 A find the values of the resistance and inductance. [60 \( \Omega \), 255 mH]

35 A circuit consisting of a resistor in series with an inductance takes 210 W at a power factor of 0.6 from a 50 V, 100 Hz supply. Find (a) the current flowing, (b) the circuit phase angle, (c) the resistance, (d) the impedance and (e) the inductance.

(a) 7 A (b) 53° lagging (c) 4.286 \( \Omega \) (d) 7.143 \( \Omega \) (e) 9.095 mH]

36 A 200 V, 60 Hz supply is applied to a capacitive circuit. The current flowing is 2 A and the power dissipated is 150 W. Calculate the values of the resistance and capacitance. [37.5 \( \Omega \), 28.61 \( \mu \)F]
16 Single-phase parallel a.c. circuits

At the end of this chapter you should be able to:

- calculate unknown currents, impedances and circuit phase angle from phasor diagrams for (a) \( R-L \) (b) \( R-C \) (c) \( L-C \) (d) \( LR-C \) parallel a.c. circuits
- state the condition for parallel resonance in an \( LR-C \) circuit
- derive the resonant frequency equation for an \( LR-C \) parallel a.c. circuit
- determine the current and dynamic resistance at resonance in an \( LR-C \) parallel circuit
- understand and calculate Q-factor in an \( LR-C \) parallel circuit
- understand how power factor may be improved

16.1 Introduction

In parallel circuits, such as those shown in Figures 16.1 and 16.2, the voltage is common to each branch of the network and is thus taken as the reference phasor when drawing phasor diagrams.

For any parallel a.c. circuit:

True or active power, \( P = VI \cos \phi \) watts (W)

or \( P = I_R^2R \) watts

Apparent power, \( S = VI \) voltamperes (VA)

Reactive power, \( Q = VI \sin \phi \) reactive voltamperes (var)

\[
\text{Power factor} = \frac{\text{true power}}{\text{apparent power}} = \frac{P}{S} = \cos \phi
\]

(These formulae are the same as for series a.c. circuits as used in Chapter 15.)

Figure 16.1

16.2 \( R-L \) parallel a.c. circuit

In the two branch parallel circuit containing resistance \( R \) and inductance \( L \) shown in Figure 16.1, the current flowing in the resistance, \( I_R \), is in-phase with the supply voltage \( V \) and the current flowing in the inductance, \( I_L \), lags the supply voltage by 90°. The supply current \( I \) is the phasor sum of \( I_R \) and \( I_L \) and thus the current \( I \) lags the applied voltage \( V \) by an angle
lying between 0° and 90° (depending on the values of $I_R$ and $I_L$), shown as angle $\phi$ in the phasor diagram.

From the phasor diagram:

$$I = \sqrt{(I_R^2 + I_L^2)} \quad \text{(by Pythagoras’ theorem)}$$

where $I_R = \frac{V}{R}$ and $I_L = \frac{V}{X_L}$

$$\tan \phi = \frac{I_L}{I_R}, \quad \sin \phi = \frac{I_L}{I} \quad \text{and} \quad \cos \phi = \frac{I_R}{I} \quad \text{(by trigonometric ratios)}$$

Circuit impedance, $Z = \frac{V}{I}$

---

**Problem 1.** A 20 Ω resistor is connected in parallel with an inductance of 2.387 mH across a 60 V, 1 kHz supply. Calculate (a) the current in each branch, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance, and (e) the power consumed.

The circuit and phasor diagrams are as shown in Figure 16.1.

(a) Current flowing in the resistor

$$I_R = \frac{V}{R} = \frac{60}{20} = 3 \text{ A}$$

Current flowing in the inductance

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi fL}$$

$$= \frac{60}{2\pi(1000)(2.387 \times 10^{-3})}$$

$$= 4 \text{ A}$$

(b) From the phasor diagram, supply current,

$$I = \sqrt{(I_R^2 + I_L^2)} = \sqrt{(3^2 + 4^2)}$$

$$= 5 \text{ A}$$

(c) Circuit phase angle,

$$\phi = \arctan \frac{I_L}{I_R} = \arctan \left( \frac{4}{3} \right) = 53.13\degree$$

$$= 53'8'' \text{ lagging}$$

(d) Circuit impedance,

$$Z = \frac{V}{I} = \frac{60}{5} = 12 \text{ Ω}$$

(e) Power consumed

$$P = VI \cos \phi = (60)(5)(\cos 53'8'') = 180 \text{ W}$$

(Alternatively, power consumed $P = I_R^2R = (3)^2(20) = 180 \text{ W}$)

---

*Further problems on R–L parallel a.c. circuits may be found in Section 16.8, problems 1 and 2, page 256.*
16.3 **R–C parallel a.c. circuit**

In the two branch parallel circuit containing resistance \( R \) and capacitance \( C \) shown in Figure 16.2, \( I_R \) is in-phase with the supply voltage \( V \) and the current flowing in the capacitor, \( I_C \), leads \( V \) by 90°. The supply current \( I \) is the phasor sum of \( I_R \) and \( I_C \) and thus the current \( I \) leads the applied voltage \( V \) by an angle lying between 0° and 90° (depending on the values of \( I_R \) and \( I_C \)), shown as angle \( \alpha \) in the phasor diagram.

From the phasor diagram:

\[
I = \sqrt{(I_R^2 + I_C^2)}, \quad \text{(by Pythagoras’ theorem)}
\]

where \( I_R = \frac{V}{R} \) and \( I_C = \frac{V}{X_C} \)

\[
\tan \alpha = \frac{I_C}{I_R}, \quad \sin \alpha = \frac{I_C}{I} \quad \text{and} \quad \cos \alpha = \frac{I_R}{I} \quad \text{(by trigonometric ratios)}
\]

Circuit impedance \( Z = \frac{V}{I} \)

---

Problem 2. A 30 \( \mu \)F capacitor is connected in parallel with an 80 \( \Omega \) resistor across a 240 V, 50 Hz supply. Calculate (a) the current in each branch, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance, (e) the power dissipated, and (f) the apparent power.

The circuit and phasor diagrams are as shown in Figure 16.2.

(a) **Current in resistor,** \( I_R = \frac{V}{R} = \frac{240}{80} = 3 \) A

Current in capacitor, \( I_C = \frac{V}{X_C} = \frac{V}{2\pi f CV} \)

\[= 2\pi f CV\]

\[= 2\pi(50)(30 \times 10^6)(240)\]

\[= 2.262 \text{ A}\]

(b) **Supply current,** \( I = \sqrt{(I_R^2 + I_C^2)} = \sqrt{(3^2 + 2.262^2)}\]

\[= 3.757 \text{ A}\]

(c) **Circuit phase angle,** \( \alpha = \arctan \frac{I_C}{I_R} = \arctan \left( \frac{2.262}{3} \right) \)

\[= 37^\circ 1' \text{ leading}\]

(d) **Circuit impedance,** \( Z = \frac{V}{I} = \frac{240}{3.757} = 63.88 \) \( \Omega \)
(c) True or active power dissipated, \( P = VI \cos \alpha \)
\[ = 240(3.757) \cos 37°1' \]
\[ = 720 \text{ W} \]
(Alternatively, true power \( P = I_R^2 R = (3)^2(80) = 720 \text{ W} \))

(f) Apparent power, \( S = VI = (240)(3.757) = 901.7 \text{ VA} \)

Problem 3. A capacitor \( C \) is connected in parallel with a resistor \( R \) across a 120 V, 200 Hz supply. The supply current is 2 A at a power factor of 0.6 leading. Determine the values of \( C \) and \( R \).

The circuit diagram is shown in Figure 16.3(a).

Power factor \( = \cos \phi = 0.6 \) leading, hence \( \phi = \arccos 0.6 \) \( = 53.13° \) leading.

From the phasor diagram shown in Figure 16.3(b),
\[ I_R = I \cos 53.13° = (2)(0.6) \]
\[ = 1.2 \text{ A} \]
and \( I_C = I \sin 53.13° = (2)(0.8) \)
\[ = 1.6 \text{ A} \]
(Alternatively, \( I_R \) and \( I_C \) can be measured from the scaled phasor diagram.)

From the circuit diagram,
\[ I_R = \frac{V}{R} \text{ from which } R = \frac{120}{1.2} = 100 \text{ } \Omega \]
and \( I_C = \frac{V}{X_C} = \frac{2\pi f CV}{C} \), from which,
\[ C = \frac{I_C}{2\pi f V} \]
\[ = \frac{1.6}{2\pi(200)(120)} \]
\[ = 10.61 \text{ } \mu\text{F} \]

Further problems on R–C parallel a.c. circuits may be found in Section 16.8, problems 3 and 4, page 256.

16.4 \( L–C \) parallel a.c. circuit

In the two branch parallel circuit containing inductance \( L \) and capacitance \( C \) shown in Figure 16.4, \( I_L \) lags \( V \) by \( 90° \) and \( I_C \) leads \( V \) by \( 90° \).

Theoretically there are three phasor diagrams possible — each depending on the relative values of \( I_L \) and \( I_C \):
(i) $I_L > I_C$ (giving a supply current, $I = I_L - I_C$ lagging $V$ by $90^\circ$)
(ii) $I_C > I_L$ (giving a supply current, $I = I_C - I_L$ leading $V$ by $90^\circ$)
(iii) $I_L = I_C$ (giving a supply current, $I = 0$).

The latter condition is not possible in practice due to circuit resistance inevitably being present (as in the circuit described in Section 16.5).

For the $L$–$C$ parallel circuit, $I_L = \frac{V}{X_L}, I_C = \frac{V}{X_C}$.

\[
I = \text{phasor difference between } I_L \text{ and } I_C, \text{ and } Z = \frac{V}{I}
\]

![Figure 16.4](image)

**Problem 4.** A pure inductance of 120 mH is connected in parallel with a 25 $\mu$F capacitor and the network is connected to a 100 V, 50 Hz supply. Determine (a) the branch currents, (b) the supply current and its phase angle, (c) the circuit impedance, and (d) the power consumed.

The circuit and phasor diagrams are as shown in Figure 16.4.

(a) Inductive reactance, $X_L = 2\pi fL = 2\pi(50)(120 \times 10^{-3})$
\[= 37.70 \ \Omega\]

Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(50)(25 \times 10^{-6})}$
\[= 127.3 \ \Omega\]

Current flowing in inductance, $I_L = \frac{V}{X_L} = \frac{100}{37.70} = 2.653 \ \text{A}$

Current flowing in capacitor, $I_C = \frac{V}{X_C} = \frac{100}{127.3} = 0.786 \ \text{A}$

(b) $I_L$ and $I_C$ are anti-phase. Hence supply current,
\[I = I_L - I_C = 2.653 - 0.786 = 1.867 \ \text{A and the current lags the supply voltage } V \text{ by } 90^\circ \text{ (see Figure 16.4(i))}\]

(c) Circuit impedance, $Z = \frac{V}{I} = \frac{100}{1.867} = 53.56 \ \Omega$

(d) Power consumed, $P = VI \cos \phi = (100)(1.867)(\cos 90^\circ)$
\[= 0 \ \text{W}\]

**Problem 5.** Repeat Problem 4 for the condition when the frequency is changed to 150 Hz.
(a) Inductive reactance, \( X_L = 2\pi(150)(120 \times 10^{-3}) = 113.1 \, \Omega \)

Capacitive reactance, \( X_C = \frac{1}{2\pi(150)(25 \times 10^{-6})} = 42.44 \, \Omega \)

Current flowing in inductance, \( I_L = \frac{V}{X_L} = \frac{100}{113.1} = 0.884 \, \text{A} \)

Current flowing in capacitor, \( I_C = \frac{V}{X_C} = \frac{100}{42.44} = 2.356 \, \text{A} \)

(b) Supply current, \( I = I_C - I_L = 2.356 - 0.884 = 1.472 \, \text{A leading} \, V \) by 90° (see Figure 4(ii))

(c) Circuit impedance, \( Z = \frac{V}{I} = \frac{100}{1.472} = 67.93 \, \Omega \)

(d) Power consumed, \( P = VI \cos \phi = 0 \, \text{W} \) (since \( \phi = 90^\circ \))

From Problems 4 and 5:

(i) When \( X_L < X_C \) then \( I_L > I_C \) and \( I \) lags \( V \) by 90°

(ii) When \( X_L > X_C \) then \( I_L < I_C \) and \( I \) leads \( V \) by 90°

(iii) In a parallel circuit containing no resistance the power consumed is zero

Further problems on \( L-C \) parallel a.c. circuits may be found in Section 16.8, problems 5 and 6, page 256.

16.5 \( LR-C \) parallel a.c. circuit

In the two branch circuit containing capacitance \( C \) in parallel with inductance \( L \) and resistance \( R \) in series (such as a coil) shown in Figure 16.5(a), the phasor diagram for the \( LR \) branch alone is shown in Figure 16.5(b) and the phasor diagram for the \( C \) branch is shown alone in Figure 16.5(c). Rotating each and superimposing on one another gives the complete phasor diagram shown in Figure 16.5(d).

The current \( I_{LR} \) of Figure 16.5(d) may be resolved into horizontal and vertical components. The horizontal component, shown as \( op \) is \( I_{LR} \cos \phi_1 \) and the vertical component, shown as \( pq \) is \( I_{LR} \sin \phi_1 \). There are three possible conditions for this circuit:

(i) \( I_C > I_{LR} \sin \phi_1 \) (giving a supply current \( I \) leading \( V \) by angle \( \phi \) — as shown in Figure 16.5(e))

(ii) \( I_{LR} \sin \phi_1 > I_C \) (giving \( I \) lagging \( V \) by angle \( \phi \) — as shown in Figure 16.5(f))

(iii) \( I_C = I_{LR} \sin \phi_1 \) (this is called parallel resonance, see Section 16.6).

There are two methods of finding the phasor sum of currents \( I_{LR} \) and \( I_C \) in Figures 16.5(e) and (f). These are: (i) by a scaled phasor diagram,
or (ii) by resolving each current into their ‘in-phase’ (i.e. horizontal) and ‘quadrature’ (i.e. vertical) components, as demonstrated in problems 6 and 7. With reference to the phasor diagrams of Figure 16.5:

Impedance of LR branch, \( Z_{LR} = \sqrt{R^2 + X_L^2} \)

Current, \( I_{LR} = \frac{V}{Z_{LR}} \) and \( I_C = \frac{V}{X_C} \)

Supply current \( I = \) phasor sum of \( I_{LR} \) and \( I_C \) (by drawing)

\[ = \sqrt{\left( I_{LR} \cos \phi_1 \right)^2 + \left( I_{LR} \sin \phi_1 \sim I_C \right)^2} \] (by calculation)

where \( \sim \) means ‘the difference between’.

Circuit impedance \( Z = \frac{V}{I} \)

\[ \tan \phi_1 = \frac{V}{R}, \sin \phi_1 = \frac{X_L}{Z_{LR}} \] and \( \cos \phi_1 = \frac{R}{Z_{LR}} \)

\[ \tan \phi = \frac{I_{LR} \sin \phi_1 \sim I_C}{I_{LR} \cos \phi_1} \] and \( \cos \phi = \frac{I_{LR} \cos \phi_1}{I} \)

Problem 6. A coil of inductance 159.2 mH and resistance 40 Ω is connected in parallel with a 30 μF capacitor across a 240 V, 50 Hz supply. Calculate (a) the current in the coil and its phase angle, (b) the current in the capacitor and its phase angle, (c) the supply current and its phase angle, (d) the circuit impedance, (e) the power consumed, (f) the apparent power, and (g) the reactive power. Draw the phasor diagram.
The circuit diagram is shown in Figure 16.6(a).

(a) For the coil, inductive reactance \( X_L = 2\pi fL \)

\[
= 2\pi(50)(159.2 \times 10^{-3})
\]

\[= 50 \Omega \]

Impedance \( Z_1 = \sqrt{R^2 + X_L^2} = \sqrt{(40^2 + 50^2)} = 64.03 \ \Omega \)

Current in coil, \( I_{LR} = \frac{V}{Z_1} = \frac{240}{64.03} = 3.748 \ \text{A} \)

Branch phase angle \( \phi_1 = \arctan \frac{X_L}{R} = \arctan \left( \frac{50}{40} \right) = \arctan 1.25 \)

\[= 51.34° = 51^\circ20' \ \text{lagging} \]

(b) Capacitive reactance, \( X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(50)(30 \times 10^{-6})} \)

\[= 106.1 \ \Omega \]

Current in capacitor, \( I_C = \frac{V}{X_C} = \frac{240}{106.1} \)

\[= 2.262 \ \text{A leading the supply voltage by 90°} \]

(see phasor diagram of Figure 16.6(b)).

(c) The supply current \( I \) is the phasor sum of \( I_{LR} \) and \( I_C \) This may be obtained by drawing the phasor diagram to scale and measuring the current \( I \) and its phase angle relative to \( V \). (Current \( I \) will always be the diagonal of the parallelogram formed as in Figure 16.6(b)).

Alternatively the current \( I_{LR} \) and \( I_C \) may be resolved into their horizontal (or 'in-phase') and vertical (or 'quadrant') components. The horizontal component of \( I_{LR} \) is

\[ I_{LR \cos(51°20')} = 3.748 \cos 51°20' = 2.342 \ \text{A} \]

The horizontal component of \( I_C \) is \( I_C \cos 90° = 0 \)

Thus the total horizontal component, \( I_H = 2.342 \ \text{A} \)

The vertical component of \( I_{LR} = -I_{LR \sin(51°20')} \)

\[= -3.748 \sin 51°20' \]

\[= -2.926 \ \text{A} \]

The vertical component of \( I_C = I_C \sin 90° \)

\[= 2.262 \sin 90° = 2.262 \ \text{A} \]
Thus the total vertical component, \( I_V = -2.926 + 2.262 = -0.664 \) A.

\( I_R \) and \( I_V \) are shown in Figure 16.7, from which,
\[
I = \sqrt{[(2.342)^2 + (-0.664)^2]} = 2.434 \text{ A}
\]
Angle \( \phi = \arctan \left( \frac{0.664}{2.342} \right) = 15.83^\circ = 15^\circ50' \) lagging

**Hence the supply current** \( I = 2.434 \) A lagging \( V \) by \( 15^\circ50' \).

**Problem 7.** A coil of inductance 0.12 H and resistance 3 kΩ is connected in parallel with a 0.02 μF capacitor and is supplied at 40 V at a frequency of 5 kHz. Determine (a) the current in the coil, and (b) the current in the capacitor. (c) Draw to scale the phasor diagram and measure the supply current and its phase angle; check the answer by calculation. Determine (d) the circuit impedance and (e) the power consumed.

The circuit diagram is shown in Figure 16.8(a).

(a) Inductive reactance, \( X_L = 2\pi f L = 2\pi(5000)(0.12) = 3770 \) Ω

Impedance of coil, \( Z_1 = \sqrt{(R^2 + X_L^2)} = \sqrt{(3)^2 + (3770)^2} \) = 4818 Ω

Current in coil, \( I_{LR} = \frac{V}{Z_1} = \frac{40}{4818} = 8.30 \text{ mA} \)

Branch phase angle \( \phi = \arctan \frac{X_L}{R} = \arctan \frac{3770}{3000} \) = 51.5° lagging
(b) Capacitive reactance, \( X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(5000)(0.02 \times 10^{-6})} \)
\[ = 1592 \, \Omega \]

Capacitor current, \( I_C = \frac{V}{X_C} = \frac{40}{1592} \)
\[ = 25.13 \, mA \text{ leading } V \text{ by } 90^\circ \]

(c) Currents \( I_{LR} \) and \( I_C \) are shown in the phasor diagram of Figure 16.8(b). The parallelogram is completed as shown and the supply current is given by the diagonal of the parallelogram. The current \( I \) is measured as \( 19.3 \, mA \text{ leading } V \text{ by } 74.5^\circ \)

By calculation, \( I = \sqrt{[(I_{LR} \cos 51.5^\circ)^2 + (I_C - I_{LR} \sin 51.5^\circ)^2]} \)
\[ = 19.34 \, mA \]

and \( \phi = \arctan \left( \frac{I_C - I_{LR} \sin 51.5^\circ}{I_{LR} \cos 51.5^\circ} \right) = 74.50^\circ \)

(d) Circuit impedance, \( Z = \frac{V}{I} = \frac{40}{19.34 \times 10^{-3}} = 2.068 \, k\Omega \)

(e) Power consumed, \( P = VI \cos \phi = (40)(19.34 \times 10^{-3})(\cos 74.50^\circ) \)
\[ = 206.7 \, mW \]

(Alternatively, \( P = I_R^2R = I_{LR}^2R = (8.30 \times 10^{-3})^2(3000) \)
\[ = 206.7 \, mW \])

Further problems on the LR–C parallel a.c. circuit may be found in Section 16.8, problems 7 and 8, page 256.

16.6 Parallel resonance and Q-factor

Parallel resonance

Resonance occurs in the two branch network containing capacitance \( C \) in parallel with inductance \( L \) and resistance \( R \) in series (see Figure 16.5(a)) when the quadrature (i.e. vertical) component of current \( I_{LR} \) is equal to \( I_C \). At this condition the supply current \( I \) is in-phase with the supply voltage \( V \).

Resonant frequency

When the quadrature component of \( I_{LR} \) is equal to \( I_C \) then: \( I_C = I_{LR} \sin \phi_1 \) (see Figure 16.9)
Hence \( V_{XC} = \left( \frac{V}{Z_{LR}} \right) \left( \frac{X_L}{Z_{LR}} \right) \), (from Section 16.5)

from which, \( Z_{LR}^2 = X_C X_L = (2\pi f, L) \left( \frac{1}{2\pi f, C} \right) = \frac{L}{C} \)  \hspace{1cm} (16.1)

Hence \( L \sqrt{(R^2 + X_L^2)} = \frac{L}{C} \) and \( R^2 + X_L^2 = \frac{L}{C} \)

Thus \( (2\pi f, L)^2 = \frac{L}{C} - R^2 \) and \( 2\pi f, L = \sqrt{\left( \frac{L}{C} - R^2 \right)} \)

and
\[
f_r = \frac{1}{2\pi} \sqrt{\left( \frac{1}{LC} - \frac{R^2}{L^2} \right)} \text{ Hz}
\]

i.e. parallel resonant frequency, \( f_r = \frac{1}{2\pi} \sqrt{\left( \frac{1}{LC} - \frac{R^2}{L^2} \right)} \text{ Hz} \)

(When \( R \) is negligible, then \( f_r = \frac{1}{2\pi \sqrt{LC}} \), which is the same as for series resonance.)

**Current at resonance**

Current at resonance, \( I_r = I_{LR} \cos \phi_1 \) (from Figure 16.9)

\[
= \left( \frac{V}{Z_{LR}} \right) \left( \frac{R}{Z_{LR}} \right) \quad \text{(from Section 16.5)}
\]

\[
= \frac{VR}{Z_{LR}^2}
\]

However from equation (16.1), \( Z_{LR}^2 = \frac{L}{C} \)

hence \( I_r = \frac{VR}{L} = \frac{VRC}{L} \) \hspace{1cm} (16.2)

The current is at a **minimum** at resonance.

**Dynamic resistance**

Since the current at resonance is in-phase with the voltage the impedance of the circuit acts as a resistance. This resistance is known as the **dynamic resistance**, \( R_D \) (or sometimes, the dynamic impedance).
From equation (16.2), impedance at resonance is given by
\[ \frac{V}{I_r} = \frac{V}{VRC} = \frac{L}{RC} \]

i.e. dynamic resistance,
\[ R_D = \frac{L}{RC} \text{ ohms} \]

Rejctor circuit

The parallel resonant circuit is often described as a rejctor circuit since it presents its maximum impedance at the resonant frequency and the resultant current is a minimum.

Q-factor

Currents higher than the supply current can circulate within the parallel branches of a parallel resonant circuit, the current leaving the capacitor and establishing the magnetic field of the inductor, this then collapsing and recharging the capacitor, and so on. The Q-factor of a parallel resonant circuit is the ratio of the current circulating in the parallel branches of the circuit to the supply current, i.e. the current magnification.

Q-factor at resonance = current magnification = \[ \frac{\text{circulating current}}{\text{supply current}} \]
\[ = \frac{I_C}{I_r} = \frac{I_{LR} \sin \phi_1}{I_r} \]
\[ = \frac{I_{LR} \sin \phi_1}{I_{LR} \cos \phi_1} = \frac{\sin \phi_1}{\cos \phi_1} \]
\[ = \tan \phi_1 = \frac{X_L}{R} \]

i.e. \[ \text{Q-factor at resonance} = \frac{2\pi f L}{R} \]

(which is the same as for a series circuit)

Note that in a parallel circuit the Q-factor is a measure of current magnification, whereas in a series circuit it is a measure of voltage magnification.

At mains frequencies the Q-factor of a parallel circuit is usually low, typically less than 10, but in radio-frequency circuits the Q-factor can be very high.

Problem 8. A pure inductance of 150 mH is connected in parallel with a 40 µF capacitor across a 50 V, variable frequency supply. Determine (a) the resonant frequency of the circuit and (b) the current circulating in the capacitor and inductance at resonance.
The circuit diagram is shown in Figure 16.10.

(a) Parallel resonant frequency, \( f_r = \frac{1}{2\pi} \sqrt{\left( \frac{1}{LC} - \frac{R^2}{L^2} \right)} \)

However, resistance \( R = 0 \). Hence,

\[
f_r = \frac{1}{2\pi} \sqrt{\left( \frac{1}{LC} \right)} = \frac{1}{2\pi} \sqrt{\left( \frac{1}{(150 \times 10^{-3})(40 \times 10^{-6})} \right)}
\]

\[
= \frac{1}{2\pi} \sqrt{\left( \frac{10^7}{(15)(4)} \right)}
\]

\[
= \frac{10^3}{2\pi} \sqrt{\left( \frac{1}{6} \right)} = 64.97 \text{ Hz}
\]

(b) Current circulating in \( L \) and \( C \) at resonance,

\[
I_{CIRC} = \frac{V}{X_C} = \frac{V}{\frac{1}{(2\pi f_r)C}} = 2\pi f_r CV
\]

Hence \( I_{CIRC} = 2\pi(64.97)(40 \times 10^{-6})(50) = 0.816 \text{ A} \)

(Alternatively, \( I_{CIRC} = \frac{V}{X_L} = \frac{V}{\frac{V}{2\pi f_r L}} = \frac{50}{2\pi(64.97)(0.15)} = 0.817 \text{ A} \))

Problem 9. A coil of inductance 0.20 H and resistance 60 \( \Omega \) is connected in parallel with a 20 \( \mu \text{F} \) capacitor across a 20 V, variable frequency supply. Calculate (a) the resonant frequency, (b) the dynamic resistance, (c) the current at resonance and (d) the circuit Q-factor at resonance.

(a) Parallel resonant frequency,

\[
f_r = \frac{1}{2\pi} \sqrt{\left( \frac{1}{LC} - \frac{R^2}{L^2} \right)}
\]

\[
= \frac{1}{2\pi} \sqrt{\left( \frac{1}{(0.20)(20 \times 10^{-6})} - \frac{(60)^2}{(0.2)^2} \right)}
\]

\[
= \frac{1}{2\pi} \sqrt{(250000 - 90000)}
\]

\[
= \frac{1}{2\pi} \sqrt{160000} = \frac{1}{2\pi}(400)
\]

\( = 63.66 \text{ Hz} \)
(b) Dynamic resistance, \( R_D = \frac{L}{RC} = \frac{0.20}{(60)(20 \times 10^{-6})} = 166.7 \ \Omega \)

(c) Current at resonance, \( I_r = \frac{V}{R_D} = \frac{20}{166.7} = 0.12 \ \text{A} \)

(d) Circuit Q-factor at resonance \( Q = \frac{2\pi f_L}{R} = \frac{2\pi(63.66)(0.2)}{60} = 1.33 \)

Alternatively, Q-factor at resonance = current magnification (for a parallel circuit) \( = \frac{I_c}{I_r} \)

\[
I_c = \frac{V}{X_c} = \frac{V}{\left(\frac{1}{2\pi f_C}\right)} = 2\pi f_C V = 2\pi(63.66)(20 \times 10^{-6})(20)
\]

\[
= 0.16 \ \text{A}
\]

Hence Q-factor \( = \frac{I_c}{I_r} = \frac{0.16}{0.12} = 1.33 \), as obtained above

---

Problem 10. A coil of inductance 100 mH and resistance 800 \( \Omega \) is connected in parallel with a variable capacitor across a 12 V, 5 kHz supply. Determine for the condition when the supply current is a minimum: (a) the capacitance of the capacitor, (b) the dynamic resistance, (c) the supply current, and (d) the Q-factor.

(a) The supply current is a minimum when the parallel circuit is at resonance.

Resonant frequency, \( f_r = \frac{1}{2\pi} \sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)} \)

Transposing for C gives: \( (2\pi f_r)^2 = \frac{1}{LC} - \frac{R^2}{L^2} \)

\[
(2\pi f_r)^2 + \frac{R^2}{L^2} = \frac{1}{LC}
\]

\[
C = \frac{1}{L \left\{ (2\pi f_r)^2 + \frac{R^2}{L^2}\right\}}
\]

When \( L = 100 \ \text{mH}, \ R = 800 \ \Omega \) and \( f_r = 5000 \ \text{Hz}, \)

\[
C = \frac{1}{100 \times 10^{-3} \left\{ 2\pi(5000)^2 + \frac{800^2}{(100 \times 10^{-3})^2}\right\}}
\]
For a particular power supplied, a high power factor reduces the current flowing in a supply system and therefore reduces the cost of cables, switch-gear, transformers and generators. Supply authorities use tariffs which encourage electricity consumers to operate at a reasonably high power factor. Industrial loads such as a.c. motors are essentially inductive (R–L) and may have a low power factor. One method of improving (or correcting) the power factor of an inductive load is to connect a static capacitor \( C \) in parallel with the load (see Figure 16.11(a)). The supply current is reduced from \( I_{LR} \) to \( I \), the phasor sum of \( I_{LR} \) and \( I_c \), and the circuit power factor improves from \( \cos \phi_1 \) to \( \cos \phi_2 \) (see Figure 16.11(b)).

### Problem 11

A single-phase motor takes 50 A at a power factor of 0.6 lagging from a 240 V, 50 Hz supply. Determine (a) the current taken by a capacitor connected in parallel with the motor to correct the power factor to unity, and (b) the value of the supply current after power factor correction.
The circuit diagram is shown in Figure 16.12(a).

(a) A power factor of 0.6 lagging means that \( \cos \phi = 0.6 \)
i.e. \( \phi = \arccos 0.6 = 53.8^\circ \)

Hence \( I_M \) lags \( V \) by 53.8° as shown in Figure 16.12(b).

If the power factor is to be improved to unity then the phase difference between supply current \( I \) and voltage \( V \) is 0°, i.e. \( I \) is in phase with \( V \) as shown in Figure 16.12(c). For this to be so, \( I_C \) must equal the length \( ab \), such that the phasor sum of \( I_M \) and \( I_C \) is \( I \).

Thus \( ab = I_M \sin 53.8^\circ = 50(0.8) = 40 \text{ A} \)

**Hence the capacitor current \( I_C \) must be 40 A for the power factor to be unity.**

(b) Supply current \( I = I_M \cos 53.8^\circ = 50(0.6) = 30 \text{ A} \)

---

**Problem 12.** A motor has an output of 4.8 kW, an efficiency of 80% and a power factor of 0.625 lagging when operated from a 240 V, 50 Hz supply. It is required to improve the power factor to 0.95 lagging by connecting a capacitor in parallel with the motor. Determine (a) the current taken by the motor, (b) the supply current after power factor correction, (c) the current taken by the capacitor, (d) the capacitance of the capacitor, and (e) the kvar rating of the capacitor.

(a) Efficiency \( \frac{\text{power output}}{\text{power input}} \) hence \( \frac{80}{100} = \frac{4800}{\text{power input}} \)

Power input = \( 4800 \div 0.8 = 6000 \text{ W} \)

Hence, \( 6000 = V I_M \cos \phi = (240)(I_M)(0.625) \),
since \( \cos \phi = \text{p.f.} = 0.625 \)

Thus current taken by the motor, \( I_M = \frac{6000}{(240)(0.625)} = 40 \text{ A} \)

The circuit diagram is shown in Figure 16.13(a).

The phase angle between \( I_M \) and \( V \) is given by:
\( \phi = \arccos 0.625 = 51.32^\circ = 51^\circ 19' \), hence the phasor diagram is as shown in Figure 16.13(b).

(b) When a capacitor \( C \) is connected in parallel with the motor a current \( I_C \) flows which leads \( V \) by 90°. The phasor sum of \( I_M \) and \( I_C \) gives the supply current \( I \), and has to be such as to change the circuit power factor to 0.95 lagging, i.e. a phase angle of \( \arccos 0.95 \) or 18°12' lagging, as shown in Figure 16.13(c).
The horizontal component of $I_M$ (shown as oa) = $I_M \cos 51^\circ 19'$

\[ = 40 \cos 51^\circ 19' \]

\[ = 25 \text{ A} \]

The horizontal component of $I$ (also given by oa) = $I \cos 18^\circ 12'$

\[ = 0.95 I \]

Equate the horizontal components gives: \[25 = 0.95 I\]

Hence the supply current after p.f. correction, $I = \frac{25}{0.95} = 26.32 \text{ A}$

(c) The vertical component of $I_M$ (shown as ab) = $I_M \sin 51^\circ 19'$

\[ = 40 \sin 51^\circ 19' \]

\[ = 31.22 \text{ A} \]

The vertical component of $I$ (shown as ac) = $I \sin 18^\circ 12'$

\[ = 26.32 \sin 18^\circ 12' \]

\[ = 8.22 \text{ A} \]

The magnitude of the capacitor current $I_C$ (shown as bc) is given by ab - ac, i.e. \[31.22 - 8.22 = 23 \text{ A}\]

(d) Current $I_C = \frac{V}{X_C} = \frac{V}{\left(\frac{1}{2\pi f C}\right)} = \frac{2\pi f CV}{1}$

From which, $C = \frac{I_C}{2\pi f V} = \frac{23}{2\pi(50)(240)} \text{ F} = 305 \mu\text{F}$

(e) kvar rating of the capacitor = \[\frac{V I_C}{1000} = \frac{(240)(23)}{1000} = 5.52 \text{ kvar}\]

In this problem the supply current has been reduced from 40 A to 26.32 A without altering the current or power taken by the motor. This means that the size of generating plant and the cross-sectional area of conductors supplying both the factory and the motor can be less — with an obvious saving in cost.

Problem 13. A 250 V, 50 Hz single-phase supply feeds the following loads (i) incandescent lamps taking a current of 10 A at unity power factor, (ii) fluorescent lamps taking 8 A at a power factor of 0.7 lagging, (iii) a 3 kVA motor operating at full load and at a power factor of 0.8 lagging and (iv) a static capacitor. Determine, for the lamps and motor, (a) the total current, (b) the overall power factor and (c) the total power. (d) Find the value of the static capacitor to improve the overall power factor to 0.975 lagging.
A phasor diagram is constructed as shown in Figure 16.14(a), where 8 A is lagging voltage V by \( \arccos 0.7 \), i.e. 45.57°, and the motor current is 3000/250, i.e. 12 A lagging V by \( \arccos 0.8 \), i.e. 36.87°

(a) The horizontal component of the currents
\[
= 10 \cos 0° + 12 \cos 36.87° + 8 \cos 45.57°
\]
\[
= 10 + 9.6 + 5.6 = 25.2 \text{ A}
\]
The vertical component of the currents
\[
= 10 \sin 0° - 12 \sin 36.87° - 8 \sin 45.57°
\]
\[
= 0 - 7.2 - 5.713 = -12.91 \text{ A}
\]
From Figure 16.14(b), total current, \( I_L = \sqrt{[(25.2)^2 + (12.91)^2]} \)
\[
= 28.31 \text{ A}
\]
at a phase angle of \( \phi = \arctan \left( \frac{12.91}{25.2} \right) \), i.e. 27.13° lagging

(b) Power factor = \( \cos \phi = \cos 27.13° = 0.890 \) lagging

(c) Total power, \( P = VI_L \cos \phi = (250)(28.31)(0.890) = 6.3 \text{ kW} \)

(d) To improve the power factor, a capacitor is connected in parallel with the loads. The capacitor takes a current \( I_C \) such that the supply current falls from 28.31 A to \( I \), lagging \( V \) by \( \arccos 0.975 \), i.e. 12.84°. The phasor diagram is shown in Figure 16.15.

\[
oa = 28.31 \cos 27.13° = I \cos 12.84°
\]
Hence \( I = \frac{28.31 \cos 27.13°}{\cos 12.84°} = 25.84 \text{ A} \)

Current \( I_C = bc = (ab - ac) \)
\[
= 28.31 \sin 27.13° - 25.84 \sin 12.84°
\]
\[
= 12.91 - 5.742
\]
\[
= 7.168 \text{ A}
\]

\[
I_C = \frac{V}{X_C} = \frac{V}{\frac{1}{2\pi f C}} = 2\pi f CV
\]

Hence capacitance \( C = \frac{I_C}{2\pi f V} = \frac{7.168}{2\pi(50)(250)} \text{ F} \)
\[
= 91.27 \mu \text{F}
\]
Thus to improve the power factor from 0.890 to 0.975 lagging a 91.27 \( \mu \text{F} \) capacitor is connected in parallel with the loads.
Further problems on power factor improvement may be found in Section 16.8 following, problems 13 to 16, page 257.

16.8 Further problems on single-phase parallel a.c. circuits

R–L parallel a.c. circuit

1 A 30 Ω resistor is connected in parallel with a pure inductance of 3 mH across a 110 V, 2 kHz supply. Calculate (a) the current in each branch, (b) the circuit current, (c) the circuit phase angle, (d) the circuit impedance, (e) the power consumed, and (f) the circuit power factor.

\[
\begin{align*}
(a) \ I_R &= 3.67 \ A, \ I_L = 2.92 \ A \ (b) \ 4.69 \ A \ (c) \ 38°30' \\
\text{lagging} \ (d) \ 23.45 \ Ω \ (e) \ 404 \ W \ (f) \ 0.783 \ \text{lagging}
\end{align*}
\]

2 A 40 Ω resistance is connected in parallel with a coil of inductance L and negligible resistance across a 200 V, 50 Hz supply and the supply current is found to be 8 A. Draw a phasor diagram to scale and determine the inductance of the coil. [102 mH]

R–C parallel a.c. circuit

3 A 1500 nF capacitor is connected in parallel with a 16 Ω resistor across a 10 V, 10 kHz supply. Calculate (a) the current in each branch, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance, (e) the power consumed, (f) the apparent power, and (g) the circuit power factor. Draw the phasor diagram.

\[
\begin{align*}
(a) \ I_R &= 0.625 \ A, \ I_C = 0.943 \ A \ (b) \ 1.13 \ A \ (c) \ 56°28' \ \text{leading} \\
(d) \ 8.85 \ Ω \ (e) \ 6.25 \ W \ (f) \ 11.3 \ VA \ (g) \ 0.55 \ \text{leading}
\end{align*}
\]

4 A capacitor C is connected in parallel with a resistance R across a 60 V, 100 Hz supply. The supply current is 0.6 A at a power factor of 0.8 leading. Calculate the value of R and C.

\[
[R = 125 \ Ω, \ C = 9.55 \ μF]
\]

L–C parallel a.c. circuit

5 An inductance of 80 mH is connected in parallel with a capacitance of 10 μF across a 60 V, 100 Hz supply. Determine (a) the branch currents, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance and (e) the power consumed.

\[
\begin{align*}
(a) \ I_C &= 0.377 \ A, \ I_L = 1.194 \ A \ (b) \ 0.817 \ A \\
(c) \ 90° \ \text{lagging} \ (d) \ 73.44 \ Ω \ (e) \ 0 \ W
\end{align*}
\]

6 Repeat problem 5 for a supply frequency of 200 Hz.

\[
\begin{align*}
(a) \ I_C &= 0.754 \ A, \ I_L = 0.597 \ A \ (b) \ 0.157 \ A \\
(c) \ 90° \ \text{leading} \ (d) \ 382.2 \ Ω \ (e) \ 0 \ W
\end{align*}
\]

LR–C parallel a.c. circuit

7 A coil of resistance 60 Ω and inductance 318.4 mH is connected in parallel with a 15 μF capacitor across a 200 V, 50 Hz supply.
Calculate (a) the current in the coil, (b) the current in the capacitor, (c) the supply current and its phase angle, (d) the circuit impedance, (e) the power consumed, (f) the apparent power and (g) the reactive power. Draw the phasor diagram.

\[ (a) 1.715 \text{ A} \quad (b) 0.943 \text{ A} \quad (c) 1.028 \text{ A at } 30^\circ 54' \text{ lagging} \]
\[ (d) 194.6 \text{ } \Omega \quad (e) 176.5 \text{ W} \quad (f) 205.6 \text{ VA} \quad (g) 105.6 \text{ var} \]

8 A 25 nF capacitor is connected in parallel with a coil of resistance 2 kΩ and inductance 0.20 H across a 100 V, 4 kHz supply. Determine (a) the current in the coil, (b) the current in the capacitor, (c) the supply current and its phase angle (by drawing a phasor diagram to scale, and also by calculation), (d) the circuit impedance, and (e) the power consumed.

\[ (a) 18.48 \text{ mA} \quad (b) 62.83 \text{ mA} \quad (c) 46.17 \text{ mA at } 81^\circ 29' \text{ leading} \quad (d) 2.166 \text{ kΩ} \quad (e) 0.683 \text{ W} \]

**Parallel resonance and Q-factor**

9 A 0.15 µF capacitor and a pure inductance of 0.01 H are connected in parallel across a 10 V, variable frequency supply. Determine (a) the resonant frequency of the circuit, and (b) the current circulating in the capacitor and inductance.

\[ (a) 4.11 \text{ kHz} \quad (b) 38.73 \text{ mA} \]

10 A 30 µF capacitor is connected in parallel with a coil of inductance 50 mH and unknown resistance \( R \) across a 120 V, 50 Hz supply. If the circuit has an overall power factor of 1 find (a) the value of \( R \), (b) the current in the coil, and (c) the supply current.

\[ (a) 37.7 \text{ } \Omega \quad (b) 2.94 \text{ A} \quad (c) 2.714 \text{ A} \]

11 A coil of resistance 25 Ω and inductance 150 mH is connected in parallel with a 10 µF capacitor across a 60 V, variable frequency supply. Calculate (a) the resonant frequency, (b) the dynamic resistance, (c) the current at resonance and (d) the Q-factor at resonance.

\[ (a) 127.2 \text{ Hz} \quad (b) 600 \text{ } \Omega \quad (c) 0.10 \text{ A} \quad (d) 4.80 \]

12 A coil of resistance 1.5 kΩ and 0.25 H inductance is connected in parallel with a variable capacitance across a 10 V, 8 kHz supply. Calculate (a) the capacitance of the capacitor when the supply current is a minimum, (b) the dynamic resistance, and (c) the supply current.

\[ (a) 1561 \text{ pF} \quad (b) 106.8 \text{ kΩ} \quad (c) 93.66 \text{ µA} \]

**Power factor improvement**

13 A 415 V alternator is supplying a load of 55 kW at a power factor of 0.65 lagging. Calculate (a) the kVA loading and (b) the current taken from the alternator. (c) If the power factor is now raised to unity find the new kVA loading.

\[ (a) 84.6 \text{ kVA} \quad (b) 203.9 \text{ A} \quad (c) 84.6 \text{ kVA} \]

14 A single phase motor takes 30 A at a power factor of 0.65 lagging from a 240 V, 50 Hz supply. Determine (a) the current taken by the
capacitor connected in parallel to correct the power factor to unity, and (b) the value of the supply current after power factor correction.

[(a) 22.80 A (b) 19.5 A]

15 A motor has an output of 6 kW, an efficiency of 75% and a power factor of 0.64 lagging when operated from a 250 V, 60 Hz supply. It is required to raise the power factor to 0.925 lagging by connecting a capacitor in parallel with the motor. Determine (a) the current taken by the motor, (b) the supply current after power factor correction, (c) the current taken by the capacitor, (d) the capacitance of the capacitor and (e) the kvar rating of the capacitor.

[(a) 50 A (b) 34.59 A (c) 25.28 A (d) 268.2 µF (e) 6.32 kvar]

16 A 200 V, 50 Hz single-phase supply feeds the following loads: (i) fluorescent lamps taking a current of 8 A at a power factor of 0.9 leading, (ii) incandescent lamps taking a current of 6 A at unity power factor, (iii) a motor taking a current of 12 A at a power factor of 0.65 lagging. Determine the total current taken from the supply and the overall power factor. Find also the value of a static capacitor connected in parallel with the loads to improve the overall power factor to 0.98 lagging. [21.74 A, 0.966 lagging, 21.68 µF]